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Calculations of the Turbulent Boundary Layer

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VARIOUS authors (see, for example, Ref. 1) have proposed transformations by which a given compressible turbulent boundary-layer problem may be reduced to a corresponding problem in incompressible flow. This enhances the importance of having available accurate calculation methods for the incompressible boundary layer, and some results of a recent survey of existing methods by the writer may be of interest at this time.

A large number of calculation methods have been applied to cases where the boundary-layer development has been measured in (nominally) two-dimensional conditions. In most cases, the step-by-step solution of the momentum integral equation, using measured H values and a skin-friction law similar to that of Ludwieg and Tillmann,[2] disagreed noticeably with the measured development of momentum thickness as shown, for example, in Figs. 1 and 2 for the data of Refs. 8 and 9. This indicates the presence of three-dimensional flows in most of the measured layers, since the neglect of the turbulence terms in the momentum equation only becomes important close to separation.

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Consequently, the various methods[3–7] that have been proposed for calculating momentum growth show no consistent relationship with experiment, since they cannot account for the range of crossflows (of either sign) that would be possible in principle for any given streamwise pressure distribution. Their relationship with the momentum integral solution suggests that a quadrature method with constants intermediate between those proposed by Spence[5] and Truckenbrodt[6] might be satisfactory in two-dimensional conditions, but the simplifying assumptions are always a source of uncertainty, and it would appear preferable to solve the integral equation directly.

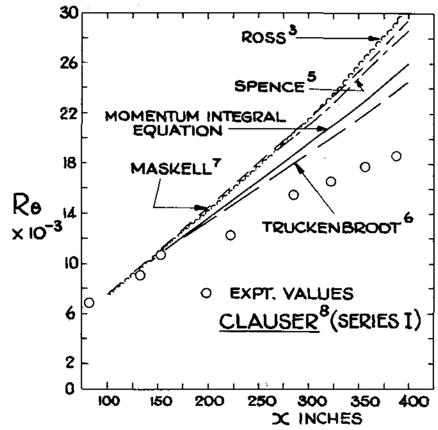

Fig. 1 Comparisons of calculated momentum thickness development.

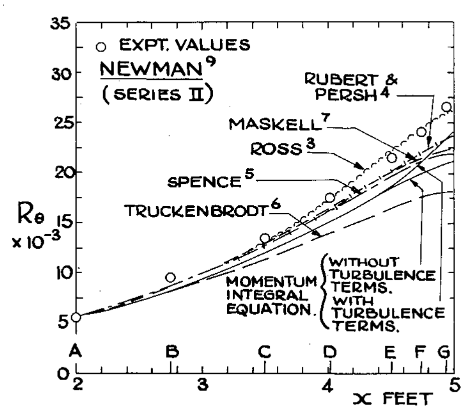

Fig. 2 Comparisons of calculated momentum thickness development.

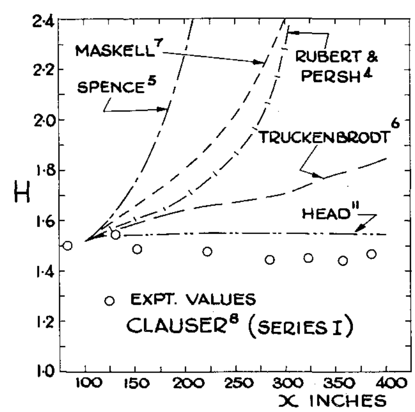

Fig. 3 Comparisons of calculated shape-factor developments.

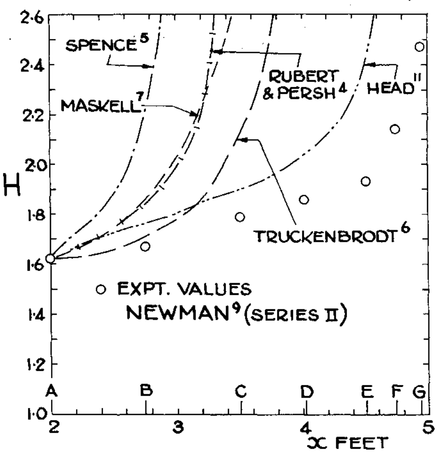

Fig. 4 Comparisons of calculated shape-factor developments.

For the calculation of shape-factor (H) development, different methods[4–7, 11] give widely differing results, as indicated in Figs. 3–5 using the measured momentum thickness development, in each case, as the basis for the calculations.

The generally rather poor predictions of some of the better known methods is clearly shown by these typical comparisons using data from Refs. 8–10. It will be seen, however, that the method of Head[11] gives the best over-all agreement with experiment and, consequently, it is recommended that this method should be used for the calculation of H, together with the momentum integral equation for the prediction of θ, the two equations being solved simultaneously step-by-step.

It is hoped that a more complete account of this work and a somewhat improved calculation method will be published at a later date.

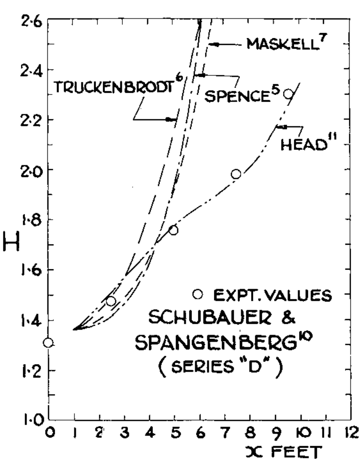

Fig. 5 Comparisons of calculated shape-factor developments.

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An Aerofoil Probe for Measuring the Transverse Component of Turbulence

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I. Introduction

At present, the accepted method of measuring instantaneously the transverse or v component of turbulent velocity employs the familiar crossed-wire probe in conjunction with dual channel hot-wire anemometer circuitry. Alternatively, for investigations where only root-mean-square values are required, a single slant wire can be used by rotating through 180° between measurements. Both of these methods involve expensive equipment and have several shortcomings. For two-point correlation work in particular, instantaneous values of v are required, which introduces the need for two crossed-wire probes and the accompanying four channels of electronics.

In an effort to circumvent this complexity a new and relatively simple probe has been developed at the Institute for Aerospace Studies. It is used with an ordinary inexpensive audio frequency amplifier. The probe consists basically of a small aerofoil and a force transducer that yields a voltage varying as the instantaneous value of v. More specifically, the aerofoil (of rectangular or circular planform) experiences a randomly varying lifting force, because of turbulent fluctuations in the flow. The aerofoil is attached to a tapered cantilever beam in which is imbedded a piezoelectric transducing element. For low values of turbulence intensity (i.e., less than 30%) the piezoelectric element produces an output voltage directly proportional to the v component of turbulent velocity.

II. Basic Theory

Figure 1 illustrates the basic principle of the aerofoil probe. We consider flow incident on the aerofoil with velocity V, at angle of attack α. In turbulent flow, V and α both vary in a random fashion. It is assumed that at any instant of time we can apply the approximation of quasi-steady linear aerofoil theory, provided that the frequency is not too high:

$$L = \tfrac{1}{2}\rho V^2 S[dC_L/d\alpha]\alpha \qquad (1)$$

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